

Sornette-Ide model for markets: Trader expectations as imaginary part

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Abstract: A nonlinear differential equation of Sornette-Ide type with noise, for a complex variable, yields endogenous crashes, preceded by roughly log-periodic oscillations in the real part, and a strong increase in the imaginary part. The latter is interpreted as the trader expectation.

Keywords: Econophysics, Langevin equations, complex variable.

Numerous microscopic models [1] for price fluctuations on stock or currency markets have been invented within the last decade [2, 3] in the physics literature. An alternative are more phenomenological differential equations for the price itself [4, 5, 6, 7, 8], with some noise as in Langevin equations. The present note follows Sornette and Ide [5] but uses a complex instead of a real variable. We interpret the real part of this complex variable z as the price (more precisely, it is proportional to the logarithm of the price in units of the fundamental price), and the imaginary part can be the trader expectation. Mathematics relates the changes in the real and imaginary parts. We will try to see crashes arising from the intrinsic market forces, with (log-periodic [9]) precursors in the real part and strong increases in the imaginary part.

The expectations of the traders in the combination of price and expectation can be defined as a two-dimensional point in the market “phase space”, which is denoted by the complex number z with $\text{Re}(z)$ = price and $\text{Im}(z)$ = expectation.

Thus, the differential equation for z as a function of time t is:

$$d^2z/dt^2 = a \cdot e^z + b \cdot (dz/dt)^3 - c \cdot z^5$$

with z at every time step changed by a small fraction $r\epsilon$ with $\epsilon \ll 1$ and a random number $-1 < r < 1$. As before [5] the c -term incorporates fundamentalist trading behaviour (buy if price is low), the b -term gives herding (buy if others buy), and $r\epsilon$ the multitude of new informations influencing the markets. Our new e^z term facilitates periodicities through the imaginary part of z which part is thus tentatively identified with trader expectations. The exponents 3 and 5 in the above equations have worked in previous simulations [7] of a real z at $a = 0$.

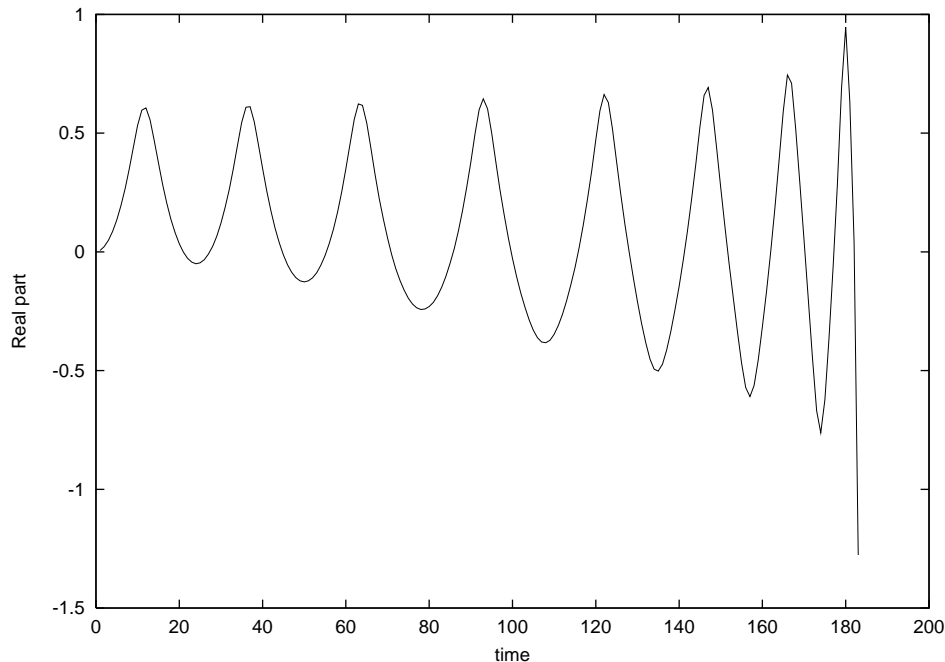


Figure 1: Real part of z versus time up to the crash.

Figures 1 and 2 show one of our many results, for $\epsilon = 0.001$, $a = 0.01$, $b = c = 1$, starting with $z = 0$, $dz/dt = (1+10^{-3}i)10^{-3}$. We see in the real part a crash preceded by oscillations of increasing amplitude and decreasing period, as in earlier work [5, 7]. The imaginary part in contrast becomes large with

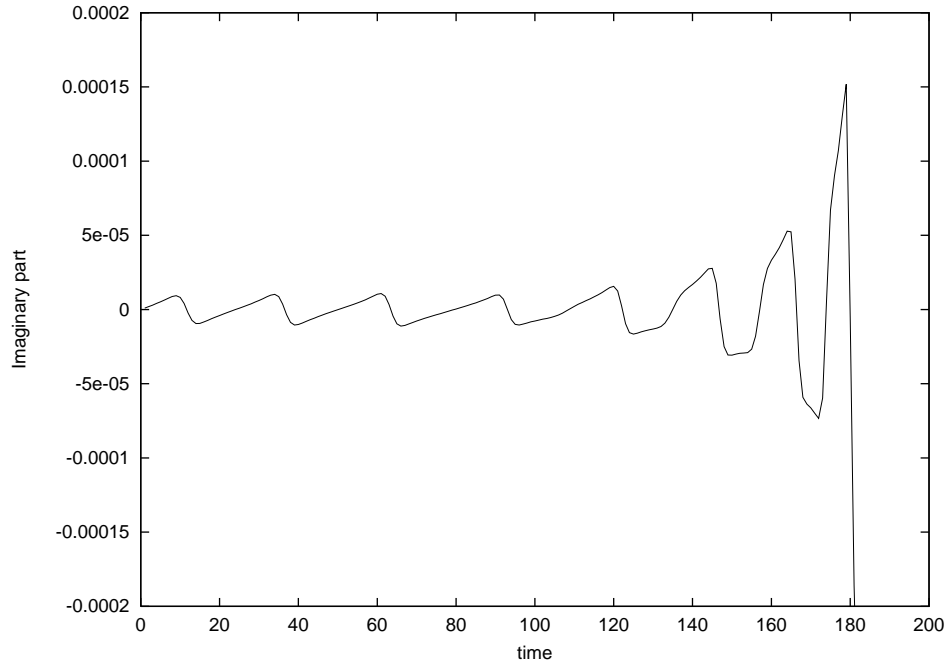


Figure 2: Same simulation as in Fig.1, but imaginary part of z is shown instead.

changing signs only shortly before the crash, as it happens with trader panic and euphoria on real markets.

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